

P. Pennanen*

Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

C. Michael†

Theoretical Physics Division, Dept. of Math. Sciences, University of Liverpool, Liverpool, UK.

The crossing from a static quark-antiquark system to a system of two static-light mesons when the separation of the static quarks is increased is calculated in zero-temperature lattice QCD. The mixing of these two states is extracted from the lattice operators. We also discuss the breaking of an excited string of a hybrid meson.

PACS numbers: 12.38.Aw 12.38.Gc 13.25.-k 13.75.Lb

I. INTRODUCTION

The breaking of a long flux tube between two static quarks into a quark-antiquark pair is one of the most fundamental phenomena in QCD. Because of its highly non-perturbative nature it has defied analytical calculation, while its large scale, e.g. when compared to the sizes of composite particles in the theory, has caused difficulties in standard nonperturbative methods. Thus string breaking has remained a widely publicized feature of the strong interaction that has never, apart from rough models, been reproduced from the theory.

String breaking can occur in hadronic decays of $Q\bar{Q}$ mesons and is especially relevant when this meson is lying close to a meson-antimeson ($Q\bar{q}Qq$) threshold. For the heaviest quarks involved in these decays applying heavy quark effective theory is a reasonable approximation.

Due to recent advances in both computational hardware and algorithms, much interest in lattice QCD has been devoted to attempts to observe string breaking. The direct approach of trying to see the flattening in the static $Q\bar{Q}$ potential at large separation has been successful only at temperatures close to the critical one [1,2]. The failure of this Wilson loop method at zero temperature seems to be mainly due to the poor overlap of the operator(s) with the $Q\bar{q}Qq$ state [3,5]. In much more easily calculable adjoint $SU(2)$ and $SU(2)$ +Higgs models without fermions a variational approach with explicit inclusion of both the Wilson loop and scalar bound state operators has worked well [3–5]. In three-dimensional $SU(2)$ with staggered fermions an improved action approach has been claimed

to be successful with just Wilson loops [6]. In QCD with fermions effective operators for $Q\bar{q}Qq$ systems are, however, hard to implement; part of the problem is the exhausting computational effort required to get sufficient statistics for light quark propagators with conventional techniques for fermion matrix inversion.

A new technique of calculating estimates of light quark propagators using Monte Carlo techniques on pseudo-fermionic field configurations [7] with maximal variance reduction [8] has been found to be very useful for systems including heavy quarks taken as static, such as single heavy-light mesons and baryons and also two heavy-light mesons [9,10]. The application of this method to the string breaking problem seems natural.

Our previous work [9,10] has concentrated on bound states of two heavy-light mesons and mechanisms of their attraction for various values of light quark isospin I_q and spin S_q . Here we continue by studying the $Q\bar{q}Qq$ system for $I_q, S_q = (0, 1)$ together with the $Q\bar{Q}$ system at distances around the string breaking point $r_b \approx 1.2$ fm, where the energies of the two systems are equal if there is no mixing.

II. QUANTUM NUMBERS AND HYBRID MESONS

When a quark and an antiquark are created from the vacuum they should have the 0^{++} quantum numbers of the vacuum. A quark-antiquark pair with $J^{PC} = 0^{++}$ has lowest orbital angular momentum $L = 1$ and is in a spin triplet. This so-called Quark Pair Creation or 3P_0 model was combined with a harmonic oscillator flux tube model by Isgur *et al.* [11] to describe local breaking (formation) of a flux tube. In our calculation the symmetries of the static approximation for the heavy quark automatically lead to only the light quark spin triplet being nonzero, which can be seen from the Dirac spin structure of the heavy-light diagram in Fig. 1.

String breaking can also occur for hybrid mesons where the gluon field between two quarks is in an excited state. Table I presents the couplings of some low-lying gluonic excitations to the quantum numbers of the resulting meson-antimeson in the static limit for the heavy quarks. In this limit CP and J_z , where z is the interquark axis,

*E-mail: petrus@hip.fi

†cmi@liv.ac.uk

are conserved. The table lists the representations of these symmetries (with Σ, Π, Δ corresponding to $J_z = 0, 1, 2$ respectively and g, u corresponding to $CP = \pm 1$). For the lowest-lying excitation, which has Π_u symmetry with $J_z = 1$, only non-zero angular momenta $L + L'$ for the resulting mesons $B_{L'}, B_L$ would be allowed, as S_q has to be zero to generate a negative CP .

Here, as for the other symmetries, $I_q = 0$ as we need the same flavour for the light quarks. The $I_q = 1$ cases do not correspond to spontaneous breaking of a string in vacuum but a correlation of a $Q\bar{Q} + q\bar{q}$ system with a $Q\bar{q} + \bar{Q}q$ system at different time, i.e. the breaking of a string in the presence of a meson. When the table is extended to nonzero L for the heavy quark plus antiquark (as opposed to $L > 0$ for a single meson) the P, C values get multiplied with $(-1)^L$. The energy levels also change; these retardation effects have been found to be relatively small [12].

Previously string breaking in hybrid mesons has been discussed from a phenomenological point of view using an extension of the approach of Isgur *et al.*, i.e. a nonrelativistic flux-tube model with decay operators from strong coupling limit of lattice gauge theory and heavy quark expansion of QCD in Coulomb gauge [13]. From this model two selection rules were given, the first one agreeing with the Π_u case in Table I; low-lying hybrids do not decay into identical mesons, the predominant channel being one s and one p -wave meson. The second rule prohibits decay of spin singlet states into only spin singlets, which is not relevant to our calculation as our heavy quark spin decouples.

An important question for hybrid meson phenomenology is the nature of the lowest state for a given set of quantum numbers at a particular heavy quark separation; a hybrid $Q\bar{Q}$ meson, a ground-state $Q\bar{Q}$ meson with a $q\bar{q}$ meson or a system of two heavy-light mesons. It is also useful to know the strength of mixing between these states. This information can be obtained, in principle, from lattice calculations and used to decide what sort of bound states are most likely to exist and what their decays will be (see also Ref. [14]).

III. LATTICE CALCULATION

We use SU(3) lattice QCD on a $16^3 \times 24$ lattice with the Wilson gauge action and the Sheikholeslami-Wohlert quark action with a nonperturbative “clover coefficient” $c_{SW} = 1.76$ and $\beta = 5.2$ with two degenerate flavours of both valence and sea quarks. The measurements were performed on 20 gauge configurations. The gauge configurations are the same as in Ref. [15] for $\kappa = 0.1395$. With these parameters we get a lattice spacing $a \approx 0.14$ fm and meson mass ratio $M_{PS}/M_V = 0.72$.

Estimators of propagators of quarks from point n to point m can be obtained from pseudofermion fields ϕ . For

each gauge configuration a sample of the pseudofermion fields is generated, and the propagators are then obtained by Monte Carlo integration [7]. Thus there is one Monte Carlo averaging for the gauge samples, and another one for the pseudofermion samples for each gauge sample. In order to reduce statistical variance of propagators a variance reduction method similar to multi-hit can be used [8]; such a reduction is essential in practice. Our variance reduction involves division of the lattice in two regions, whose boundary is kept fixed while the ϕ -fields inside are replaced by their multi-hit averages. We use 24 pseudofermionic configurations per each gauge configuration.

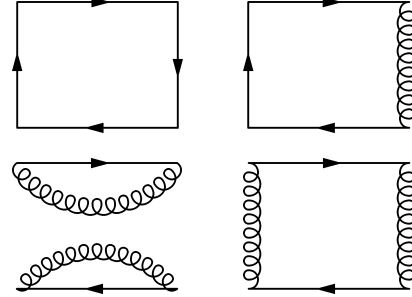


FIG. 1. Diagrams involved in the calculation; the Wilson loop W , the heavy-light correlator U , the unconnected meson-antimeson D and the box diagram B .

Figure 1 shows the diagrams involved in the calculation with the time axis in the horizontal direction. The solid lines are heavy quark propagators, which in the static approximation are just products of gauge field variables. The wiggly lines are light quark propagators, obtained essentially as a product of pseudofermionic variables from each end which have to be in different variance reduced regions [8].

In the large T limit both the quark-antiquark and two-meson operators should in principle approach $e^{-E_0(R)T}$ with $E_0(R)$ being the ground state of the system. In practice the Wilson loop has a very small overlap with the two-meson state, which leads to great practical difficulties in observing the flattening $E_0(R) \rightarrow 2M_{Q\bar{q}}$ from it at large R . The heavy-light term U is necessary to obtain the correct ground state by explicitly including both quark-antiquark and two-meson states, and allows us to measure their overlap, which is crucial for string breaking to happen.

To estimate the ground (and excited) state energy of our observables we always use a variational basis formed from different degrees of spatial *fuzzing* of the operators. This allows the use of moderate values of T instead of the infinite time limit to reduce excited state contributions. The resulting correlation matrix $C(R, T)$ is then diagonalised to get the eigenenergies.

Due to the variance reduction method dividing the lattice in two halves in the time direction, the box diagram

and the heavy-light correlator in Fig. 1 have to be turned “sideways” on the lattice; i.e., the time axis in the diagrams is taken to be one of the spatial axes to keep the light quark propagators going from one variance reduced volume to another. This induces technical complications that greatly increase the memory and CPU demands of the measurement program.

For two flavours the $I_q = 0$ wavefunction is of the form $(u\bar{u} + d\bar{d})/\sqrt{2}$, which gives factors of 1, $\sqrt{2}$, 2 for D , U , B respectively. For light quark spin we get the triplet states as in Ref. [9].

In this first study we concentrate on the ground state breaking, i.e. the first row of Table I. Investigation of the hybrid meson breaking requires diagrams not included in Fig. 1, which involve the hybrid $Q\bar{Q}$ and $Q\bar{Q} + \bar{q}q$ operators. We estimate that for the Π_u excited state the excited string breaking happens in the same distance range as for the ground state due to the non-zero momenta of the resulting mesons (masses taken from Ref. [8]), which makes it harder to obtain sufficient accuracy as the spatial operators for excitations involve subtractions rather than sums of lattice paths.

IV. RESULTS

A. Variational approach

A full variational matrix involving the Wilson loop, heavy-light correlator and the $Q\bar{q}Qq$ correlators gives the ground and excited state energies and corresponding operator overlaps as a function of heavy quark separation, in analogue to the approach of Refs. [4,5] for the adjoint string breaking and Refs. [3] for the $SU(2)$ +Higgs model. We use a local light quark creation (annihilation) operator and an extended version where a fuzzed path of link variables with length two separates the operator from the heavy quark line. For the link variables involved in the $Q\bar{Q}$ operators we have two fuzzing levels. The two $Q\bar{Q}$ and three $Q\bar{q}Qq$ basis states then give a 5×5 correlation matrix $C(R, T)$ that can be diagonalised. However, for our present statistics the full matrix gives a reasonable signal only for $r < r_b$.

In Figure 2 the results from a calculation using just the most fuzzed basis states for both $Q\bar{Q}$ and $Q\bar{q}Qq$ (a 2×2 matrix) are shown. At r_b we would expect the ground and excited state energies to be separated by twice the mixing coefficient x (see below). We observe a larger separation which is presumably due to our statistics not being sufficient to give accurate plateaus for the energies.

Although this full variational approach is in principle the most direct way to study string breaking, we find that it is possible to focus on string breaking explicitly, as we now discuss.

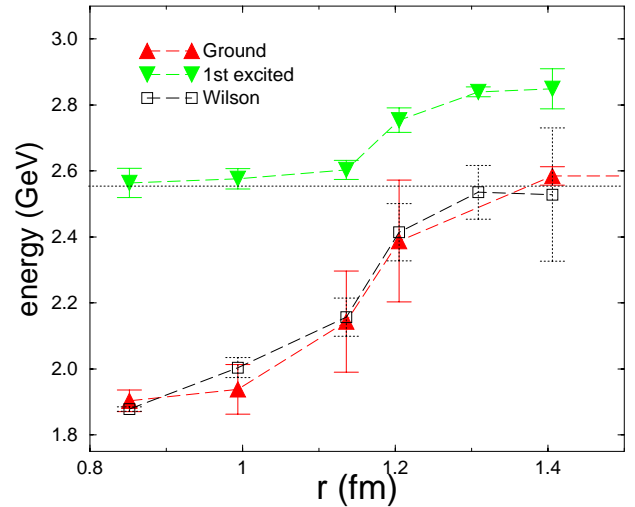


FIG. 2. Ground and excited state from a variational calculation including $Q\bar{Q}$ and $Q\bar{q}Qq$ operators. The highest fuzzed basis state for both is used here. The ground state of the Wilson loop and $2m_{Q\bar{q}}$ are also shown.

B. Mixing matrix element

In full QCD, there is mixing of energy levels between states coupling to Wilson lines (flux tube) and $Q\bar{q}Qq$ states. To get the mixing matrix element the correlation between a Wilson line and a $Q\bar{q}Qq$ operator has to be considered. In order to study the operator mixing from this heavy-light correlator one needs to use results (energies and couplings) from both diagonal operators separately: thus from the Wilson loop (with ground state contribution given by $W(T) = w^2 \exp[-V(R)T]$) and the unconnected $Q\bar{q}Qq$ correlator (eg. $D(T) = d^2 \exp[-M(R)T]$ from the ground state) where we use a variational basis to suppress excited states.

The ground state contribution to the heavy-light correlator can then be written as

$$U(T) = x(R) \sum_{t=0}^T w e^{-V(R)t} e^{-M(R)(T-t)} d + O(x^3) \quad (1)$$

In the quenched case the contributions from fermion loops inside the correlator are absent, removing the $O(x^3)$ terms in Eq. 1. The box term is expressed in the same manner as

$$B(T) = x^2(R) \sum_{t_1=0}^T \sum_{t_2 \geq t_1}^T d e^{-V(R)t_1} e^{-M(R)(t_2-t_1)} \times e^{-V(R)(T-t_2)} d + O(x^4) \quad (2)$$

The operator mixing coefficient x for the $Q\bar{Q}$ and $Q\bar{q}Qq$ states can be extracted from these expressions. Near the string breaking point (where $V(R) = M(R)$), in the infinite time limit, only the ground state contributions survive. We use

$$x = \frac{U(T)}{\sqrt{W(T)D(T)}} \frac{f^{T/2}}{1 + \dots + f^T} + O(x^3) \quad (3)$$

$$= \sqrt{\frac{B(T)}{D(T)}} \frac{f^{T/2}}{\sqrt{1 + \dots + (T+1)f^T}} + O(x^2) \quad (4)$$

The factors of $f \equiv \exp(V(R) - M(R))$ account for departures from the string breaking point.

In the quenched case there is no mixing between the energy levels of the quark-antiquark and two-meson systems, and x can be extracted using Eqs. 3,4. As $x \ll 1$ the non-leading terms in the expressions for x are small and we may use also these formulas with our unquenched data - with a resulting decrease in errors compared to the full variational study of the preceeding subsection.

Our assumption about neglecting excited state contributions can be tested by obtaining consistent results for x from both relations for several T values. To improve further on our estimate of x we diagonalise separately W , D and B to enhance the ground state contributions and use the first two diagonalisations to extract the ground state of U . Our results with bootstrap errors can be seen in table II. Assuming constant x for $0.99 \text{ fm} \leq r \leq 1.31 \text{ fm}$ gives us a best estimate of $x/a = 0.033(6)/a = 46(8) \text{ MeV}$. This is about half of the value of $x = 100 \text{ MeV}$ obtained using a strong coupling mixing model and the experimental $\Upsilon(4S)$ decay rate [16].

Our analysis shows that the string breaking matrix element is small but non-zero. We are able, however, to find two independent ways to estimate it (using all four diagrams in Fig. 1) and we obtain $x = 46(8) \text{ MeV}$ with light quarks that are around the strange quark mass. This is the first non-perturbative determination from QCD of the string breaking matrix element. Because of its small value, direct observation of string breaking from the spectrum is difficult to achieve.

Acknowledgement

We thank A.M. Green and K. Rummukainen for discussions. Some of the calculations, consuming 70 GB of disk and 2×10^{16} FLOPs, were performed with the excellent resources provided by the CSC in Espoo, Finland.

- [1] **CP-PACS** Collaboration, S. Aoki *et. al.*, Nucl. Phys. Proc. Suppl. **73**, 216 (1999), [hep-lat/9809185](#).
- [2] E. Laermann, C. DeTar, O. Kaczmarek and F. Karsch, Nucl. Phys. Proc. Suppl. **73**, 447 (1999), [hep-lat/9809105](#).
- [3] O. Philipsen and H. Wittig, Phys. Rev. Lett. **81**, 4056 (1998), [hep-lat/9807020](#); **ALPHA** Collaboration, F. Knechtli and R. Sommer, Phys. Lett. **B440**, 345 (1998), [hep-lat/9807022](#).
- [4] C. Michael, Nucl. Phys. Proc. Suppl. **26**, 417 (1992).
- [5] P. W. Stephenson, Nucl. Phys. **B550**, 427 (1999), [hep-lat/9902002](#); O. Philipsen and H. Wittig, Phys. Lett. **B451**, 146 (1999), [hep-lat/9902003](#); P. de Forcrand and O. Philipsen, [hep-lat/9912050](#).

- [6] H. Trottier, Phys. Rev. **D60**, 034506 (1999), [hep-lat/9812021](#).
- [7] G. M. de Divitiis, R. Frezzotti, M. Masetti and R. Petronzio, Phys. Lett. **B382**, 393 (1996), [hep-lat/9603020](#).
- [8] **UKQCD** Collaboration, C. Michael and J. Peisa, Phys. Rev. **D58**, 034506 (1998), [hep-lat/9802015](#).
- [9] **UKQCD** Collaboration, C. Michael and P. Pennanen, Phys. Rev. **D60**, 054012 (1999), [hep-lat/9901007](#).
- [10] **UKQCD** Collaboration, P. Pennanen, C. Michael and A. M. Green, [hep-lat/9908032](#).
- [11] N. Isgur, R. Kokoski and J. Paton, Phys. Rev. Lett. **54**, 869 (1985).
- [12] K. J. Juge, J. Kuti and C. J. Morningstar, [hep-lat/9909165](#).
- [13] P. R. Page, E. S. Swanson and A. P. Szczepaniak, Phys. Rev. **D59**, 034016 (1999), [hep-ph/9808346](#).
- [14] C. Michael, Proceedings of Heavy Flavours 8, Southampton, UK (1999), [hep-lat/9911219](#).
- [15] **UKQCD** Collaboration, C. R. Allton *et. al.*, Phys. Rev. **D60**, 034507 (1999), [hep-lat/9808016](#).
- [16] I. T. Drummond and R. R. Horgan, Phys. Lett. **B447**, 298 (1999), [hep-lat/9811016](#).

J_z	CP	Gluon field symmetry	$\Sigma_{1g} + q\bar{q}$ state	I_q	S_q	$L + L'$
0	+	Σ_{1g}	ω	0	1	0
1	-	Π_u	h	0	1	
0	-	Σ_{1u}	η	0	0	
1	+	Π_g	ω	1	0	
2	+	Δ_g	f_2	1	1	
2	-	Δ_u	η_2	0	2	
0	-	-	π	1	0	0
0	+	-	ρ	1	0	

TABLE I. Relation of gluonic excitations of a hybrid meson to ground-state properties of the meson pair resulting from string breaking, in the static limit for the heavy quarks. The last three columns refer to the quantum numbers of the light quarks in the meson-antimeson system.

$r \text{ (fm)}$	Eq. 3 without $O(x^3)$	Eq. 4 without $O(x^2)$
0.85	0.040(3)	0.081(8)
0.99	0.025(4)	0.045(5)
1.14	0.025(3)	
1.31	0.031(15)	0.040(25)

TABLE II. Operator mixing x extracted using a variational approach with two different formulas. The values are taken at $T = 4$ and have bootstrap errors. Physical dimensions are obtained by multiplying with $a^{-1} = 1.39 \text{ GeV}$